

Rescattering effects and the determination of the gluon density for $x \ll 1$ ^a

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We consider the possible role of rescattering effects in the determination of the gluon density for the LHC from DIS data. We discuss a method that uses results of s-channel calculations to estimate these effects, and comment on potential applications to diffractive and multi-parton interactions.

The Large Hadron Collider will operate with very high gluon luminosities. Production processes initiated by gluons contribute a great many events to a number of cross sections of primary interest for the LHC physics program. Reliable predictions for these cross sections depend on the determination of the gluon density in the proton and its accuracy.^{1,2,3,4,5} As parton luminosities rise steeply for decreasing momentum fraction x , a large number of events sample the gluon density at $x \ll 1$.

It has long been known that the theoretical accuracy of gluon-density determinations for $x \ll 1$ is affected by higher-loop $\ln(1/x)$ corrections to QCD evolution equations. See e.g. Ref.⁶ for early numerical investigations. The study of these corrections motivates current searches for evolution schemes (see Refs.^{7,8,9}, and references therein) that incorporate the resummation of $\ln(1/x)$ contributions at the next-to-leading-logarithmic accuracy.^{10,11} An improved theoretical control on the $x \ll 1$ region is expected from the inclusion of these terms.

Because the DIS data used at present to extract the gluon density for $x < 10^{-2}$ do not have very high Q^2 , it is natural to ask whether non-negligible effects on the theoretical accuracy may also come from corrections that are suppressed by powers of $1/Q^2$ but are potentially enhanced as $x \rightarrow 0$. These could affect the determination of the gluon density f_g primarily through a contribution δ to the Q^2 -derivative of the F_2 structure function,

$$\frac{dF_2}{d\ln Q^2} \simeq P_{qg} \otimes f_g [1 + \delta] + \text{quark term} \quad , \quad \delta \simeq \sum_{k \geq 1} a_k (\alpha_s \frac{1}{x^\nu} \frac{\Lambda^2}{Q^2})^k \quad . \quad (1)$$

Here P_{qg} is the perturbative gluon-to-quark evolution kernel, and the correction δ arises from multi-parton correlation terms in the operator product expansion,

$$F_2 = C \otimes f + \frac{1}{Q^2} C^{(4)} \otimes f^{(4)} + \dots \quad . \quad (2)$$

The enhanced $x \rightarrow 0$ behavior in Eq. (1) can be produced from graphs with multiple gluon scatterings, and is consistent with observations of approximate geometric scaling in low- x data¹² and with models for saturation.^{13,14,15}

Standard methods to take account of multiple scatterings are s-channel methods (see e.g. the lectures in Ref.¹⁵), essentially orthogonal to those of the parton picture. The basic degrees of freedom in the s-channel picture are described by correlators of eikonal Wilson lines — at the simplest level, two-point correlators, interpretable as color dipoles. To identify the correction

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from rescattering graphs to the parton result in Eq. (1), a sufficiently precise “dictionary” is needed to connect the two pictures. Ref. ¹⁶ presents an approach to analyze this connection.

The method is based on constructing explicitly an s-channel representation for the renormalized parton distribution function in terms of Wilson-line matrix elements, convoluted with light-cone wave functions. In this representation the quark distribution f_q is given by the coordinate-space convolution

$$xf_q(x, \mu) = \int d\mathbf{z} \int d\mathbf{b} u(\mu, \mathbf{z}) \Xi(\mathbf{z}, \mathbf{b}) , \quad (3)$$

where Ξ is the hadronic matrix element of eikonal-line operators,

$$\begin{aligned} \Xi(\mathbf{z}, \mathbf{b}) &= \int [dP'] \langle P' | \frac{1}{N_c} \text{Tr} \{ 1 - V^\dagger(\mathbf{b} + \mathbf{z}/2) V(\mathbf{b} - \mathbf{z}/2) \} | P \rangle , \\ V(\mathbf{z}) &= \mathcal{P} \exp \left\{ -ig \int_{-\infty}^{+\infty} dz^- A_a^+(0, z^-, \mathbf{z}) t_a \right\} , \end{aligned} \quad (4)$$

\mathbf{z} is the transverse separation between the eikonal lines, \mathbf{b} is the impact parameter, and the function $u(\mu, \mathbf{z})$ is evaluated explicitly in Ref. ¹⁶ at one loop using the $\overline{\text{MS}}$ scheme for the renormalization of the ultraviolet divergences $\mathbf{z} \rightarrow 0$.

The representation (3), once evaluated in a well-prescribed renormalization scheme, is the key ingredient that allows one to relate ^{16,17} results of s-channel calculations for structure functions to the OPE factorization (2), and, in particular, to identify the power-suppressed corrections arising from the multiple gluon scatterings, treated in the high-energy approximation of Eq. (4). These corrections are found to depend on moments of Ξ , schematically in the form

$$\frac{dF_2}{d \ln Q^2} = \left(\frac{dF_2}{d \ln Q^2} \right)_{\text{LP}} + \sum_{n=1}^{\infty} R_n \frac{\lambda^2(n)}{(Q^2)^n} , \quad (5)$$

where the first term in the right hand side is the leading-power parton result, and the λ^2 in the subleading terms are given by the analytically continued moments

$$\lambda^2(-v) = \frac{1}{\Gamma(v)} \int \frac{d\mathbf{z}}{\pi \mathbf{z}^2} (\mathbf{z}^2)^{v-1} \int d\mathbf{b} \Xi(\mathbf{z}, \mathbf{b}) . \quad (6)$$

The coefficients R_n are evaluated to order α_s , as functions of x and $\ln Q^2$, from the lightcone wave functions, while the moments $\lambda^2(n)$ are dimensionful nonperturbative parameters, to be determined from comparison with experimental data.

In practice, the usefulness of the result in Eqs. (5),(6) comes from the fact that the hadronic matrix element Ξ can be related by a short-distance expansion for $\mathbf{z} \rightarrow 0$ to a well-prescribed integral of the gluon distribution function ¹⁶. Then the moments λ^2 can be parameterized in terms of the factorization/renormalization scales at which the gluon distribution and the strong coupling are evaluated. These scales are to be taken of the order of the inverse mean transverse distance $1/|\mathbf{z}|$, and can be tuned to the data. The result of doing this for F_2 data ¹⁸ at both low and high Q^2 is shown in Fig. 1 in the left hand side plot. ¹⁷ The corresponding power correction in Eq. (5) is plotted on the right hand side of Fig. 1. Here we show the correction normalized to the full answer and multiplied by -1 , using CTEQ parton distributions. ¹⁹

Fig. 1 indicates that with physically natural choices of the parameters in the nonperturbative matrix elements (4) one can achieve a sensible description of data for $x < 10^{-2}$ in a wide range of Q^2 and still have moderate power corrections to $dF_2/d \ln Q^2$. Corrections turn out to be negative and below 20 % for $x \gtrsim 10^{-4}$ and $Q^2 \gtrsim 1 \text{ GeV}^2$. However, Fig. 1 also indicates that for small x the corrections fall off slowly in the region of medium Q^2 , $Q^2 \simeq 1 - 10 \text{ GeV}^2$, behaving effectively like $1/Q^\lambda$ with λ close to 1. ¹⁷ For instance, one has $\lambda \simeq 1.2$ for the curve $x = 10^{-3}$

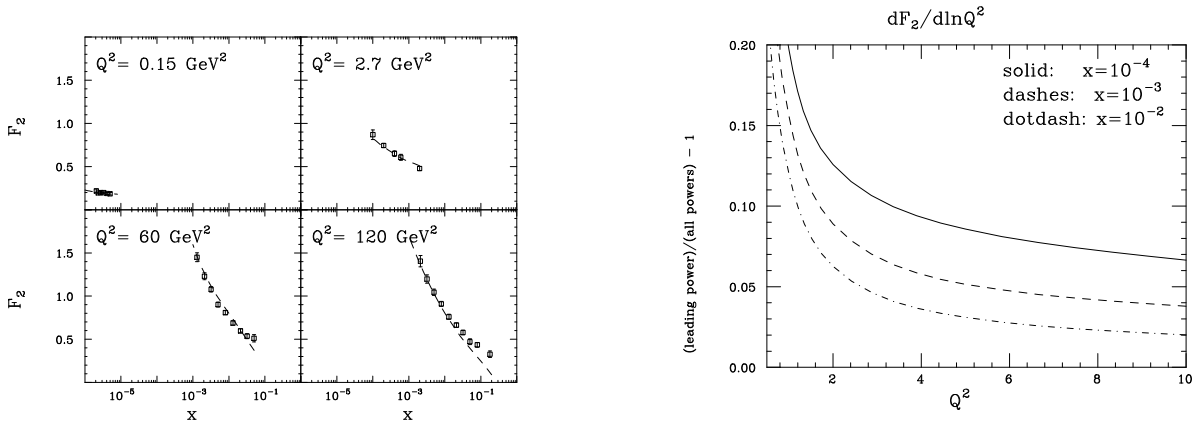


Figure 1: (left) The result of fitting the λ^2 parameters to the data [18]; (right) power corrections to $dF_2/d\ln Q^2$ versus Q^2 at different values of x .

in the right-hand side plot of Fig. 1. As a consequence of the slowly decreasing behavior, the power corrections stay on the order of 10% up to Q^2 of a few GeV^2 for $x \lesssim 10^{-3}$. This slow fall-off differs from parameterizations of higher twist commonly used in global analyses (see e.g. Ref. ³), and may be relevant for phenomenology as it affects the medium Q^2 region of the data used to extract f_g at low x .

Corrections larger than for F_2 are found for the longitudinal component F_L .¹⁷ This provides additional motivation for the forthcoming F_L measurements²⁰, as well as fits²¹ investigating power-like terms in F_L (at both high and low x). We observe also that Fig. 1 is obtained using NLO parton distributions, and the decrease in the low- x gluon at NNLO² could be consistent with the possibility that NNLO parton distributions correspond to smaller power corrections. However, the detailed interpretation of this behavior will be subtle, as distinctly different dynamics drive the power-like and NNLO effects, unlike the high- x case in the analyses^{3,22}.

It is worth emphasizing that the above results depend on the validity of the high-energy approximation and s-channel representation (3), and the perturbation expansion for $u(\mu, z)$. The rationale for this expansion lies with the dynamical cut-off on large distances z imposed by unitarity requirements (“black disc” limit) on the correlator Ξ .^{14,15,16} But the size of this cut-off at collider energies is difficult to determine. The highest sensitivity to it may come from measurements of the diffractive part of the DIS cross section.^{23,24,25} In this case the s-channel representation is bilinear in Ξ . The comparison²⁶ of diffractive data with theoretical predictions based on diffractive parton distributions indicates that the dynamical cut-off lies at substantially higher momenta for color-octet eikonal-line matrix elements than for color-triplet. That is, gluons’ shadow is stronger. See e.g. Ref. ²⁷ for a recent discussion. In diffractive DIS this can be linked^{23,28} to the distinctive pattern of the observed scaling violation²⁴ and detailed features of the associated jet distributions.^{25,29,30} More generally, it suggests that the expansion used is better justified for processes directly coupled to the gluon distribution than for F_2 , see e.g. applications to F_L (or its diffractive component³¹) and jet final states. A critical discussion, including the quark case, is given in Ref. ¹⁶.

Note that the question of how to perform QCD calculations that incorporate multiple scatterings along with perturbative evolution becomes especially compelling in the case of Monte Carlo event generators³². The application considered above deals with corrections to Q^2 evolution, i.e., a picture based on strongly ordered k_\perp ’s. But the method relies on high-energy approximations that may be better suited for extension to evolution with ordering in energies, or angles. It could thus more likely be adapted to the modeling of multiparton processes in Monte Carlo generators^{33,34} based on high-energy evolution equations.

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